ИЗПИТНА ТЕМА

по дисциплината "Висша математика II" КТТ, КСТ

Задачи

1. Намерете екстремумите на функцията $y=\frac{1}{x}+2\,arctg\,x$. $y' = ?$

2. Намерете инфлексните точки на функцията $y=e^{-x^2}$. $y'' = ?$

3. Пресметнете $\int\frac{x^2+2}{\sqrt{x}}dx$. $\sqrt{x}=t$; $x=t^2$ $dx = 2t\,dt$

4. Пресметнете $\int_0^{\pi/2} x\cos 2x\,dx$. интегриране по части

Теория

5. Сходимост на числова редица

6. Свойства на определения интеграл.

① $y = \frac{1}{x} + 2\,\mathrm{arctg}\,x$

$$y' = -\frac{1}{x^2} + \frac{2}{1+x^2} = \frac{-1-x^2+2x^2}{x^2(1+x^2)} =$$

$$= \frac{x^2-1}{x^2(1+x^2)}, \quad y' = 0 \Rightarrow x^2 - 1 = 0$$

$$x = \pm 1$$

знаци на $y' \Rightarrow$ $\quad + \quad -1 \quad - \quad 1 \quad +$

$$y_{max} = y(-1) = \frac{1}{-1} + 2\,\mathrm{arctg}(-1) = -1 - 2\frac{\pi}{4} = -1 - \frac{\pi}{2}$$

$$y_{min} = y(1) = \frac{1}{1} + 2\,\mathrm{arctg}\,1 = 1 + \frac{\pi}{2}$$

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② Инфлексни точки: $y'' = 0$

$$y = e^{-x^2} \Rightarrow y' = -2x\,e^{-x^2}$$

$$y'' = (-2x)'e^{-x^2} + (-2x)\left(e^{-x^2}\right)' = -2e^{-x^2} \cancel{+ 4x^2 e^{-x^2}} + 4x^2 e^{-x^2} =$$

$$= e^{-x^2}(4x^2 - 2), \quad y'' = 0 \Rightarrow 4x^2 - 2 = 0$$

$$x^2 = \frac{2}{4} = \frac{1}{2} \Rightarrow x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$$

$$y = e^{-\left(\frac{\sqrt{2}}{2}\right)^2} = e^{-1/2}$$

(3) $\int \frac{x^2+2}{\sqrt{x}}dx =$ Полагаме $\sqrt{x}=t$, $x=t^2$

$dx = 2t\,dt$

$= \int \frac{t^4+2}{\cancel{t}} \cdot 2\cancel{t}\,dt = 2\int t^4 dt + 4\int dt =$

$= 2\frac{t^5}{5} + 4t = \frac{2}{5}(\sqrt{x})^5 + 4\sqrt{x} + C$

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(4) $\int_0^{\pi/2} x\cos 2x\,dx =$ интегриране по части $=$

$= \frac{1}{2}\int_0^{\pi/2} x\cos 2x\,d2x = \frac{1}{2}\int_0^{\pi/2} x\,d\sin 2x = \underbrace{\frac{x}{2}\sin 2x\Big|_0^{\pi/2}}_{0} -$

$- \frac{1}{2}\int_0^{\pi/2}\sin 2x\,dx = -\frac{1}{4}\int_0^{\pi/2}\sin 2x\,d2x =$

$= \frac{1}{4}\cos 2x\Big|_0^{\pi/2} = \frac{1}{4}\cos\pi - \frac{1}{4}\cos 0 = -\frac{1}{4} - \frac{1}{4} = \underline{\underline{-\frac{1}{2}}}$